Modelling and Applications in Mathematics Education

The 14th ICMI Study

International Commission on Mathematical Instruction

Edited by

Werner Blum, Peter L. Galbraith, Hans-Wolfgang Henn and Mogens Niss

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PREFACE

Among the themes that have been central to mathematics education during the last 30 years are those of mathematical *modelling* and *applications* of mathematics to extra-mathematical fields. More generally we refer to these as *relations between mathematics and the extra-mathematical world* (sometimes also called the "real world") or preferably, according to Henry Pollak, the "rest of the world". That applications and modelling have been important themes in mathematics education can be inferred from the wealth of literature on these topics, including material generated from a multitude of national and international *conferences*. In particular let us mention firstly the ICMEs (the International Congresses on Mathematical Education), with their regular working or topic groups and lectures on applications and modelling; and secondly the series of ICTMAs (the International Conferences on the Teaching of Mathematical Modelling and Applications) which have been held biennially since 1983. Their Proceedings and Survey Lectures, have addressed the state-of-the-art at the relevant time, and contain many examples, studies, conceptual contributions and resources involving relations between the real world and mathematics, for all levels of the educational system. In *curricula and textbooks* we find today many more references to real world phenomena and problems than, say, twenty years ago. Yet while applications and modelling play more important roles in many countries' classrooms than in the past, there still exists a substantial gap between the ideals expressed in educational debate and innovative curricula on the one hand, and everyday teaching practice on the other. In particular, genuine modelling activities are still rather rare in mathematics classrooms.

Altogether, during the last few decades there has been considerable work in mathematics education that has centred on applications and modelling. Many activities have had a primary focus on practice, e.g. construction and trial of mathematical modelling examples for teaching and examination purposes, writing of application-oriented textbooks, implementation of applications and modelling in existing curricula, or development of innovative, modelling-oriented curricula. Several of these activities also contain research components such as: clarification of relevant concepts; investigation of competencies and identification of difficulties and strategies activated by students when dealing with application problems; observation and analysis of teaching; study of learning and communication processes in modelling-oriented lessons; and evaluation of alternative approaches used to assess performance in applications and modelling. In particular during the last ten years the number of genuine research contributions has increased considerably.

That applications and modelling have been, and continue to be, central themes in mathematics education is not at all surprising. Nearly all questions and problems in mathematics education, that is questions and problems concerning human learning and the teaching of mathematics, influence and are influenced by relations between mathematics and some aspects of the real world. For instance, one essential answer (albeit not the only one) to the question as to why all persons ought to learn mathematics is that it provides a means for understanding the world around us, for coping with everyday problems, or for preparing for future professions. When addressing the question of how individuals acquire mathematical knowledge, we cannot avoid the role of its relationship to reality, especially the relevance of situated learning (including the problem of the dependence of learning on specific contexts). General questions as to what "mathematics" is, as a part of our culture and as a social phenomenon, of how mathematics has emerged and developed, involve also "applications" of mathematics in other disciplines, in nature and society. Today mathematical models and modelling have penetrated a great variety of disciplines, leaving only a few fields (if any) where mathematical models do not play some role. This increasing involvement has been substantially supported and accelerated by the availability of powerful electronic tools, such as calculators and computers, with their enormous communication capabilities.

Relations between the real world and mathematics are particularly relevant within the current OECD (Organisation for Economic Co-operation and Development) PISA project. What is being tested in PISA (Programme for International Student Assessment), is mathematical literacy, that is, according to the PISA framework, "an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage in mathematics, in ways that meet the needs of that individual's life as a constructive, concerned, and reflective citizen." That means the emphasis in PISA is on the use of mathematical knowledge in a multitude of situations and contexts. In several countries, this project has initiated an intense discussion about aims and design of mathematics instruction in schools, and especially about the role of mathematical modelling, applications of mathematics and relations to the real world. Such deliberations are also occurring in countries outside the OECD.

This book is the Study Volume of ICMI Study 14 on "Applications and Modelling in Mathematics Education", which began effectively in 2002 with the development of the Discussion Document by the Programme Committee (published in Educational Studies in Mathematics 51(2002)1/2, pp 149-171). In mounting this Study, ICMI has taken into account the reasons mentioned above for the importance of relationships between mathematics and the real world, as well as the contemporary state of the educational debate, and of research and development in this field. This does not, of course, mean that we claim to know satisfactory answers to the essential questions in this area, and that the role of the Study is simply to provide a forum for putting these together. Rather, an important aim of the Study and this Volume has been to identify shortcomings, as well as to stimulate further research and development activities, in addition to reporting on existing research and practice.

Documenting the state-of-the-art in a field and identifying deficiencies and needed research requires a structuring framework. This is particularly important in an area which is as complex and difficult to survey as the teaching and learning of mathematical modelling and applications. As we have seen, this topic not only deals with most of the essential aspects of the teaching and learning of mathematics at large, it also touches upon a wide variety of versions of the real world outside mathematics that one seeks to model. Perceived in this way, the topic of applications and modelling may appear to encompass all of mathematics education plus much more. It is evident, therefore, that we need a way of conceptualising the field so as to reduce complexity to a meaningful and tractable level. That is why this Volume commences with an introductory Part I where we clarify some of the basic concepts and notions of the field, and offer a conceptualisation that helps to structure it and to identify important challenges and questions. This introductory part, at the same time, provides a concise access to the field for the uninitiated reader together with a brief sketch of its history.

Following from this introductory part, the Volume contains plenary papers given at the Study Conference (Dortmund, February 2004) and various papers that address important issues in the field. It is stressed, however, that this Study Volume is not simply the Proceedings of the Study Conference – rather, the production of this Volume has involved an independent process. Of course, the papers presented at the Study Conference provided a rich source for this Volume, and the majority of papers here were derived in some way from those Conference papers. However, many of the papers in this Volume have been produced independently of the Study Conference, in particular to fill gaps that became obvious during the Conference.

We would like to express our sincere thanks to the members of the Programme Committee for this Study who have contributed in various ways to producing this Volume. In particular, several members have acted as editors of Sections in this Volume. Without their work and devotion, this extensive Volume could not have been completed. Our thanks go equally to all the authors who have contributed to this Volume and thus helped to make it – so we hope – a rich source of information and inspiration for readers. We also thank ICMI very much for having given priority to this Study, and in par-

ticular its Secretary, Bernard Hodgson, for his sensitive way of channelling ICMI views and proposals into this Study while, at the same time, leaving the organisers and editors with all the freedom they wanted and needed to undertake this task. Eventually, we would like to thank the Publisher, Springer, also for their patience when the completion of this Volume was on their agenda.

Let us finish this Preface by expressing our hope that this ICMI Study 14 Volume will be of value both for mathematics educators, mathematics teachers and mathematicians as well as for interested professionals in other disciplines in which mathematics plays an essential role, and that it will contribute to a strengthening and further development of the field of applications and modelling in mathematics education, and to an intensification of various kinds of research and practice activities in the field.

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Part 1

INTRODUCTION

Part 1

INTRODUCTION

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Abstract: In this part of the volume, we shall give an introduction both to the field of applications and modelling in mathematics education and to the present volume. In section 1, we present the field of applications and modelling to the mathematics educator who is not a specialist in the field. In Section 2, we explain the basic terms, notions and distinctions in applications and modelling. On this basis, we provide, in Section 3, the conceptualisation of the field adopted in this ICMI Study. This conceptualisation is centred on a number of issues which will be the subject of Section 4. In Section 5, we briefly outline the historical development of applications and modelling in mathematics education. Finally, in Section 6, the structure and organisation of the present book will be described and explained.

1. INTRODUCTION FOR THE UN-INITIATED READER

Our endeavour in this section is to briefly present the field of applications and mathematical modelling in mathematics education to interested mathematics educators who are not specialists in the field.

For the remainder of this section, we need a first terse definition of the basic concepts involved. An *application* of mathematics occurs every time mathematics is *applied*, for some purpose, to deal with some domain of the *extra-mathematical world*, for instance in order to understand it better, to investigate issues, to explain phenomena, to solve problems, to pave the way for decisions, etc. The extra-mathematical world can be another subject or discipline, an area of practice, a sphere of private or social life, etc. The term "real world" is often used to describe the world outside mathematics, even though, say, quantum physics or orbitals in chemistry may appear less than real to some. The extra-mathematical world is then a helpful way of indicating that part of the wider "real world" that is relevant to a particular issue or problem. In any application of mathematics a mathematical *model* is involved, explicitly or implicitly. A mathematical model consists of the extra-mathematical domain, D, of interest, some mathematical domain M, and a mapping from the extra-mathematical to the mathematical domain (see Fig. 1-1). Objects, relations, phenomena, assumptions, questions, etc. in D are identified and selected as relevant for the purpose and situation and are then mapped – translated – into objects, relations, phenomena, assumptions, questions, etc. pertaining to M. Within M, mathematical deliberations, manipulations and inferences are made, the outcomes of which are then translated back to D and interpreted as conclusions concerning that domain. This so-called *modelling cycle* may be iterated several times, on the basis of validation and evaluation of the model in relation to the domain, until the resulting conclusions concerning D are satisfactory in relation to the purpose of the model construction. The term *modelling* refers to the entire process, and everything involved in it – from structuring D, to deciding upon a suitable mathematical domain M and a suitable mapping from D to M, to working mathematically within M, to interpreting and evaluating conclusions with regard to D, and to repeating the cycle several times if needed or desirable.

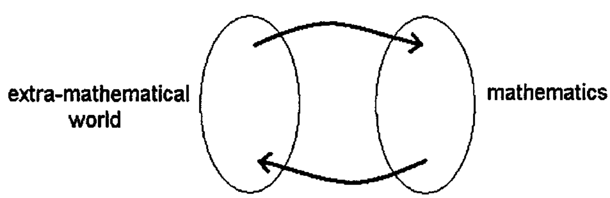

Figure 1-1. Mathematics and the rest of the world

At various times in the history of mathematics teaching and learning, it has been debated whether some forms of mathematical applications (or, in more recent times, modelling) should have a place in different sorts of mathematics curricula, or whether the extra-mathematical utilisation of mathematics should be the responsibility of those subjects that utilise the applications and the modelling. Sometimes – or in some places – curricula have focused on pure mathematics while leaving applications and modelling (if relevant) to other subjects. At other times – or in other places – curricula have made explicit room for applications and modelling. The very fact that there are, from time to time, such debates about the possible place and role of applications and modelling in the teaching and learning of mathematics suggests that there are issues to consider and think about. If we suppose that it has been agreed that applications and modelling should play some part in the mathematical education of a given category of students, hosts of further questions arise that are to do with "why?", "under what circumstances and conditions?", "what?", "when?", "how?", "taught by whom?", and so on. All of these questions need to be dealt with by research as well as by practice.

When it comes to the question of "why?" there exists a fundamental *duality* (not to be mistaken for a dichotomy) between the categories of possible answers. The first category focuses on "applications and modelling *for the learning of mathematics*", i.e. on the actual or potential ways in which applications and modelling may be a vehicle for facilitation and support of students' learning of mathematics as a subject. The other category focuses on "learning mathematics so as to *develop competency in applying mathematics and building mathematical models*" for areas and purposes that are basically extra-mathematical. We are dealing with a duality because the relationship between mathematical learning, and applications and modelling, has two different orientations, depending on which is the goal and which is the means. This duality plays out very differently at different educational levels, and for different types of curricula.

At the primary and lower secondary levels the duality is only seldom made explicit, as it is quite customary at these levels to insist on both orientations simultaneously, recognising that they are intrinsically intertwined. A major reason why we teach mathematics to typical students at primary or lower secondary level, is that they should become able to use mathematics in a variety of contexts and situations outside the classroom. This implies that applications and modelling should be on the agenda of teaching and learning, without necessarily using these terms. On the other hand, for most primary or lower secondary students it is difficult to motivate or learn mathematical concepts, methods, techniques, terminology, and results and to engage in mathematical activity, unless clear reference is being established to the use and relevance of mathematics to extra-mathematical contexts and situations, which are often also responsible for creating meaning and sense-making with regard to the mathematical entities at issue.

Also at further educational levels where mathematics is being taught for vocational or professional purposes, that are closely related to other areas or subjects, for instance in, say, carpentry, plumbing, banking, economics or engineering, the duality is sometimes deliberately kept implicit so as to blur what is the goal and what is the means of mathematics teaching. However, even though implicit, the duality is there nevertheless, provided of course, there is such a thing as "mathematics" explicitly mentioned in the curriculum.

In contrast, at upper secondary or tertiary level the duality between "applications and modelling for the learning of mathematics" and "learning mathematics for applications and modelling" is indeed, often, a significant one, worth further exploration.

Let us take a closer look at the two poles of this duality. Firstly, the "applications and modelling for the learning of mathematics" pole is to do with (a) demonstrating to students that mathematics is actually being used by people outside the mathematics classroom for a variety of reasons and purposes, thus helping to generate a richer image of the nature and role of mathematics; (b) helping to provide meaning and interpretation to mathematical entities and activities; and (c) – partly as a consequence of (a) and (b) – providing motivation for students to engage in the study of mathematics by helping to shape their beliefs and attitudes towards it. The second pole, "learning mathematics for applications and modelling", focuses (a) on one goal of the teaching and learning of mathematics, namely to equip students with the capability to bring mathematics to bear outside itself; and (b) on the fact that the extraneous use of mathematics is always brought about through mathematical models and modelling.

In principle one might think that even if an ultimate goal is to foster applications and modelling capabilities with students, this would not necessarily require applications and modelling to be dealt with in the classroom. In fact, from time to time, there has been a tendency amongst mathematics educators, including teachers, to assume that once someone has learnt theoretical mathematics in a proper and efficient way and to a satisfactory extent, that individual will be able to apply mathematics in other areas and contexts without further teaching. Hence there is, according to this view, no reason to spend precious time in the study of mathematics on dealing with applications and modelling. Moreover, the reasoning goes, that as the extraneous use of mathematics involves by definition, non-mathematical objects, phenomena, features, and facts, the mathematics classroom is not the right place to deal with such matters, particularly since the mathematics teacher more often than not would be an amateur in dealing with them, and hence by and large be unqualified for the task. Instead, to the extent that mathematics is relevant in other areas and subjects, the teachers and professionals in those areas and subjects, will master the mathematics involved well enough for them to be in charge of the applications and modelling themselves.

However, there is, today, ample evidence from practice and research that there is *no automatic transfer* from having learnt purely theoretical mathematics to being able to use it in situations that have not already been fully mathematised. Moreover, even if mathematics is being activated within other areas or subjects there is evidence that those aspects of models and modelling that are to do with the relationships between the mathematical representations and the domain of application, including validation of model assumptions and results, are not taken seriously. This suggests that if we want stu-